# Analysis of the ICE Combiner for Multiple Antenna Arraying

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The passage of the International Cometary Explorer (ICE) through the tail of comet Giacobini–Zinner took place on September 11, 1985, at approximately 11:04 GMT. The signal-to-noise ratio of the data received from the ICE spacecraft during the comet encounter was improved by arraying the 64-m antenna channels A and B (RCP and LCP) with the two 34-m antennas. Specially designed combiners were built to combine the signals received by the three antennas at the different DSN sites to ensure that the spacecraft's weak signal was received. Although the ICE spacecraft was built with a 5-W transmitter and with a small antenna designed to provide data from no farther than 1 million miles, these combiners provided enough signal margin during the encounter to receive the ICE transmitted data from within the tail of the comet Giacobini–Zinner, 44 million miles from Earth.

## I. Introduction

ICE telemetry link analysis showed that reception from the ICE at the Giacobini–Zinner encounter at the desired 1024-bps data rate would be made feasible by combining the two downlinks (channels A and B) received by the DSN 64-m antenna [1]. The addition of two 34-m antennas to this array would provide an additional margin of 0.8 dB to the received signal and would reduce effects caused by weather, etc. (see Table 1).

Five combiners were built for the DSN to support the capability to array together the telemetry output of the 34-m antennas at DSS-12 and DSS-15 and the 64-m antenna RCP and LCP channels. The combiners were multi-input-port (two, three, or four port), single- or triple-output-port devices to allow operational flexibility for ICE operations and testing (Fig. 1).

## II. The Combiners

The function of the combiners is to accept baseband signals from the telemetry phase detector's output; to weight the signals appropriately in accordance with their respective SNRs to provide the maximum signal-to-noise ratio at the output; to sum the signals; and to provide the combined output signal to prime and backup telemetry data streams. The block diagram in Fig. 2 shows how this was accomplished. Baseband signals were passed through variable attenuators which were used to set the weighting factors to provide the maximum signal-to-noise ratio at the output of the summing amplifier. Current amplifiers were used to provide the combined output signal for prime and backup telemetry data streams.

Table 1 lists the relevant parameters for the four-input-port combiner for the ICE–GZ encounter.

## III. ICE Encounter Support

Two combiners were operated at each DSN complex during the encounter, the prime unit combining all available signals and a backup combining only the two strong signals from the 64-m antenna (Fig. 2).

Baseband receiver telemetry phase detector outputs were patched directly into an input port, and the output of the combiners was patched directly to a telemetry string consisting of a Subcarrier Demodulator Assembly (SDA), a Symbol Synchronizer Assembly (SSA), and a Telemetry Processor Assembly (TPA).

The combiner required external monitoring to validate its performance. (Separate telemetry strings provided assessments of proper performance.) The SSA measured signal-to-noise ratios, and the sequential decoder provided a symbol error rate statistic. Thus, when array testing was under way, all available telemetry strings were used to measure configuration setup conditions and long-term performance.

Baseband signals were combined using optimum combining coefficients $\{\alpha_k\}$ so that the output SNR was the sum of the input SNRs. Appendix A describes the expression used for combining coefficients $\{\alpha_k\}$.

For the ICE-GZ encounter, the combining coefficients were normalized so that the total output power from the combiners was approximately the same as the baseband power levels ($\approx$6 dBm), to be within the acceptable range for the SDAs:

$$\{\alpha_i\} = \alpha_i \, k; \qquad k = \frac{1}{\Sigma \alpha_i}$$

The arrival time difference for signals at the SPC-10 (DSS-14/15/12) combiners has two main components. One component is the transport delay from the receiver input on the antenna to the input port of the combiner. The transport delay for DSS-14 and DSS-15 is approximately 1.3 $\mu$s and can be ignored. The transport delay for DSS-14 and DSS-12 is approximately 55 $\mu$s, since the stations are 16.5 km apart. Because the ICE combiner does not compensate for this delay,

the result is 0.2 dB loss in SNR when a signal arrives at each antenna at the same time.

The second component is the geometric time-of-arrival difference for the baseline between the DSS-14 and DSS-12 antennas. For the ICE-GZ encounter, DSS-12 will contribute about 0.3 dB to the net symbol signal-to-noise ratio (SSNR) from spacecraft rise to meridian crossing, but its contributions will deteriorate to nearly 0.1 dB by spacecraft set. This expectation was supported by observation during the test pass on July 26, 1985, at a bit rate of 1024 bps.

For the time near comet encounter, the arrival time delay for DSS-12 to DSS-14 followed approximately the following:

| Spacecraft Position | T (DSS-12/DSS-14) |
|---|---|
| Rise | 6 $\mu$s |
| Meridian | 12 $\mu$s |
| Set | -35 $\mu$s |

Appendix B derives the expected SNR degradation due to arrival time dephasing during the Giacobini–Zinner comet encounter.

## IV. Conclusions

Figure 3 shows the observed performance at Goldstone using the three- and four-input-port combiners for this time interval. The actual improvement in signal-to-noise ratio realized by the ICE resistive combiners can be estimated from the plots of the symbol error rate (SER) and the SSNR performed on DOY 185 (1985) using the three-input-port combiner to combine DSS-14 (channels A and B) with DSS-12. These plots are shown in Fig. 4.

At this time, the ICE spacecraft was operating at a bit rate of 512 bps. The sum of the DSS-14 channels A and B, with SSNRs of 1.3 dB and -0.2 dB, yields a theoretical combined SSNR of 3.68 dB, while the observed SSNR was 3.6 dB. Thus, the actual improvement due to antenna arraying using the ICE resistive combiner was within 0.1 dB of the calculated prediction. The combiner is currently being used to support Pioneer and ICE spacecraft.

# Reference

[1] N. A. Fanelli, L. Efron, and R. J. Muellerschoen, "ICE Second Halley Radial: TDA Mission Support and DSN Operations," TDA Progress Report 42-87, vol. July–September 1986, Jet Propulsion Laboratory, Pasadena, California, pp. 285–290, November 15, 1986.

**Table 1. Symbol definitions and values**

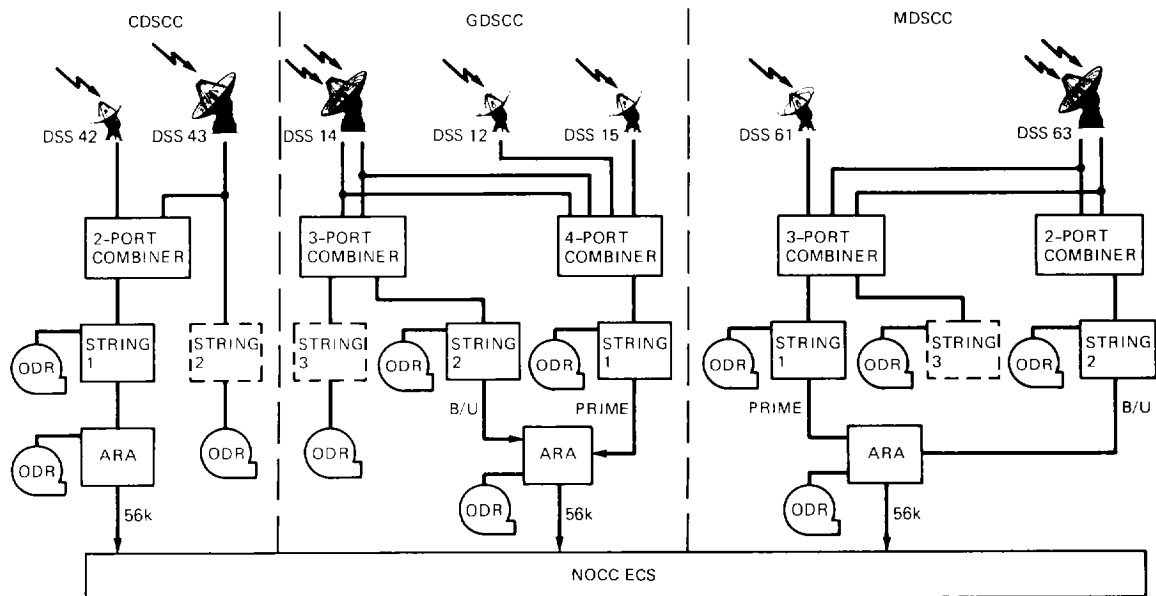| Symbol | Description | Value |
|---|---|---|
| $R_1$ | SSNR at DSS-14 (RCP) | 0 dB |
| $R_2$ | SSNR at DSS-14 (LCP) | −1 dB |
| $R_3$ | SSNR at DSS-12 | −7 dB |
| $R_4$ | SSNR at DSS-15 | −8 dB |
| $r_s$ | Symbol rate | 2048 sps |
| $B_{rf}$ | Baseband bandwidth | 730 kHz |
| $\alpha_1$ | DSS-14 (RCP) | 0.52 |
| $\alpha_2$ | DSS-14 (LCP) | 0.48 |
| $\alpha_3$ | DSS-12 | 0.16 |
| $\alpha_4$ | DSS-15 | 0.14 |
| $R_1 + R_2$ | Combined SSNR (DSS-14 only) | 2.55 dB |
| $R_1 + R_2 + R_3 + R_4$ | Combined SSNR (DSS-14, 12, 15) | 3.34 dB |

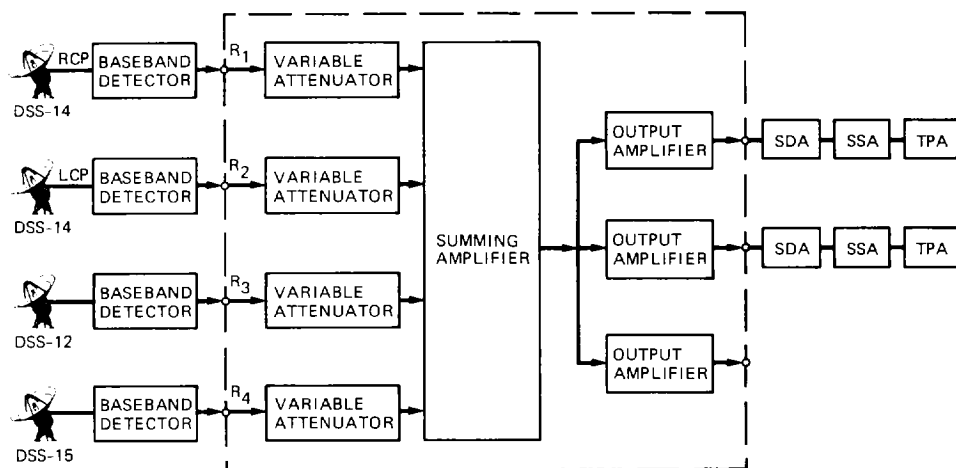**Fig. 1. ICE encounter support configuration**



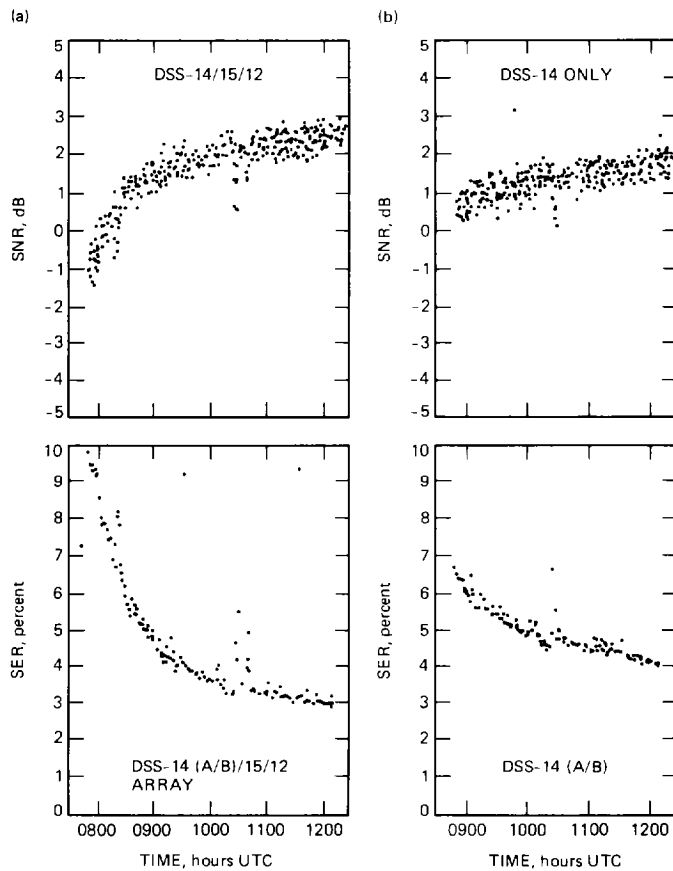**Fig. 2. Four-input-port combiner block diagram**

Fig. 3. Observed ICE link performance at encounter (comet tail crossing = 1104 Z on DOY 254): (a) four-port combiner; (b) three-port combiner
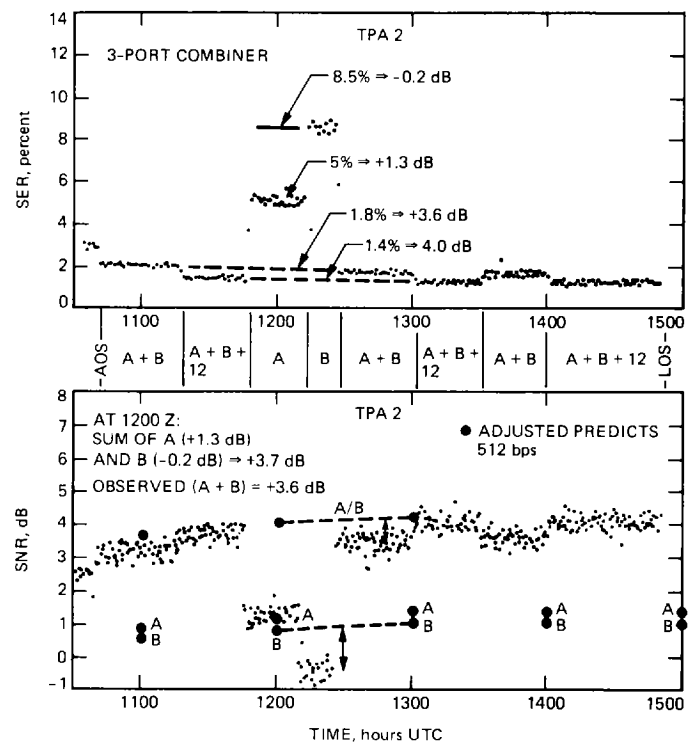


Fig. 4. Performance of Goldstone at 512 bps on 1985 DOY 185

# Appendix A
# Arraying Methodology

If we have $n$ Gaussian voltage sources with means $u_K$ and standard deviation $\sigma_K$, we define the signal-to-noise power ratio $R_K$ as

$$R_K = \frac{u_K^2}{2\sigma_K^2} \qquad K = 1, \ldots, n \qquad (A1)$$

where $u_K$ is the mean integrated symbol voltage, $\sigma_K$ is the rms integrated symbol noise voltage, and the channel total power is

$$P_K = u_K^2 + \sigma_K^2 \qquad (A2)$$

These Gaussian sources model the integrated symbol voltage distribution from each of $n$ telemetry chains with independent noise

$$\rho_K(x) = \frac{1}{\sigma_K \sqrt{2\pi}} \exp\left\{ -\frac{1}{2} \left( \frac{x \pm u_K}{\sigma_K} \right)^2 \right\} \qquad (A3)$$

signal voltage density. Now, if we combine the signals with $n$ combining coefficients $\{\alpha_K\}$, we get, for signal voltages that add coherently:

$$u_A = \sum_{K=1}^{n} \alpha_K u_K \qquad (A4)$$

for noise powers that add incoherently:

$$\sigma_A^2 = \sum_{K=1}^{n} \alpha_K^2 \sigma_K^2 \qquad (A5)$$

Then

$$R_A = \frac{u_A^2}{2\sigma_A^2} \qquad (A6)$$

is the combined array SNR and

$$P_A = u_A^2 + \sigma_A^2 \qquad (A7)$$

is the combined output channel total power.

The optimum combining ratios $\{\alpha_K\}$ to maximize $R_A$ with respect to the $\{\alpha_K\}$ are found by taking $\partial_{RA}/\partial_{\alpha K} = \phi$.

Thus:

$$\frac{u_A}{\sigma_A^2} \frac{\partial_{uA}}{\partial \alpha_K} - \frac{u_A^2}{2\sigma_A^2} \frac{1}{\sigma_A^2} \frac{\partial \sigma_A^2}{\partial \alpha_K} = \phi \qquad (A8)$$

And using (A2) and (A3), we get

$$\alpha_K = \left( \frac{u_K}{u_A} \right) \left( \frac{\sigma_A^2}{\sigma_K^2} \right) \left( \frac{u_A}{u_K} \right) \left( \frac{R_K}{R_A} \right) \qquad (A9)$$

From (A1) and (A2), we get

$$u_K = \sqrt{\frac{2 P_K R_K}{1 + 2R_K}} \qquad (A10)$$

From (A5) and (A6), we get

$$u_A = \sqrt{\frac{2 P_A R_A}{1 + 2R_A}} \qquad (A11)$$

Thus:

$$\alpha_K = \frac{R_K}{R_A} \sqrt{\frac{P_A R_A (1 + 2 R_K)}{P_K R_K (1 + 2 R_A)}} = \sqrt{\frac{P_A R_K (1 + 2 R_K)}{P_K R_A (1 + 2 R_A)}} \qquad (A12)$$

In the case where $P_K = P_A$, we get

$$\alpha_K = \sqrt{\frac{R_K (1 + 2 R_K)}{R_A (1 + 2 R_A)}} \qquad \text{(A13)}$$

Using these values for $\{\alpha_K\}$, we have

$$R_A = \frac{u_A^2}{2\sigma_A^2} = \frac{\left(\sum_{K=1}^{n} \alpha_K\right)^2}{2 \sum_{K=1}^{n} \alpha_K^2 \sigma_K^2}$$

$$= \sum_{K=1}^{n} \frac{u_K^2}{2\sigma_K^2} = \sum_{K=1}^{n} R_K \qquad \text{(A14)}$$

So for the above choice of $\{\alpha_K\}$, the output SNR is the sum of the input SNRs. Note that the combined SNR $R_A$ is the same if all $\{\alpha_K\text{'s}\}$ are multiplied by some constant value.

In particular, we can normalize so the largest $\alpha_K$ is equal to unity $\bar{\alpha}_K = \alpha_K / \alpha_R ; \alpha_R = \max \{\alpha_K\}$.

# Appendix B

# Degradation Due to Dephasing

During the Giacobini-Zinner comet encounter, the ICE spacecraft was operating at a bit rate $(r_B)$ of 1024 bps with a rate 1/2 code and biphase modulation format.

The symbol period $T_s = 1/r_s$ where $r_s$ is the symbol rate where $r_s = 2r_b$. The Manchester transition period $T_x$ is

$$T_x = \frac{1}{2} T_s = \frac{1}{4r_B} \qquad (B1)$$

where $r_B = 1/T_B$.

Now let $\rho_{xs}$ be the symbol transition probability and let $\rho_{xm}$ be the Manchester transition probability.

Thus,

$$\rho_{xm} = 1 - \frac{\rho_{xs}}{2} \qquad (B2)$$

so the average number of transitions per second $N_x$ is

$$N_x = \frac{\rho_{xm}}{T_x} = 4\,\rho_{xm}\,r_B \qquad (B3)$$

for PN symbols $(\rho_{xs} \simeq 0.5)$ $N_x \simeq 3r_B$.

With no dephasing, the combined output signal $u_A(0)$ becomes:

$$u_A(0) = \sum_K u_K \qquad (B4)$$

and the in-phase power

$$u_A^2(0) = \sum_K \sum_1 u_K u_1 \qquad (B5)$$

Now let any of the $K$th channels be dephased by $\Psi_K$ seconds (see Fig. B1). Then the combined output signal voltage $u_A(\Psi_K)$ will be:

$$u_A(\Psi_K) = \sum_K u_K (1 - 2N_x\,|\Psi_K|) \quad \text{where } K = 1, \ldots, n \qquad (B6)$$

while the output noise power $\sigma_A^2$ is unaffected.

$$u_A(\Psi_K) = u_A(0) - 2N_x \sum_K u_K\,|\Psi_K|$$

$$= u_A(0)\left[1 - 2N_x \sum_K \frac{u_K}{u_A(0)}\,|\Psi_K|\right] \qquad (B7)$$

Thus:

$$u_A(\Psi_K) = u_A(0)\,\eta_A(\Psi_K) \qquad (B8)$$

where

$$\eta_A(\Psi_K) = \left[1 - 2N_x \sum_K \frac{u_K}{u_A(0)}\,|\Psi_K|\right] \qquad (B9)$$

Using $\ln(1 - x) \simeq -x$ when $x \ll 1$, then the amplitude loss in nats of the combined output signal becomes

$$\ln \eta_A(\Psi_K) \simeq 2N_x \sum_K \frac{u_K}{u_A(0)}\,|\Psi_K| \qquad (B10)$$

and the power loss

$$\ln \eta\rho(\Psi_K) = 2\ln \eta_A(\Psi_K) \simeq 4N_x \sum_K \frac{u_K}{u_A(0)}\,|\Psi_K| \qquad (B11)$$

In the case where $2N_x \, \Psi_K \, \alpha_K \, u_K \, / \, u_A \ll 1$ is not true, then we have:

$$u_A(\Psi_K) = u_A(0) \left(1 - 2 N_x \, \Psi_K \, \frac{\alpha_K \, u_K}{u_K}\right)$$

$$= u_A(0) \left(1 - 2 N_x \, \Psi_K \, \frac{u_K^2 \, \sigma_A^2}{u_A^2 \, \sigma_K^2}\right)$$

(B12)

and from Eq. B9 we have

$$u_A(\Psi_K) = u_A(0) \left(1 - 2 N_x \, \Psi_K \, \frac{R_K}{R_A}\right)^2 = u_A^2(0) \cdot X$$

(B13)

Thus, the SNR degradation is given by $X$.

Example: An array configuration with no dephasing and a bit rate $(r_B)$ of 1024 bps has a theoretical combined output SSNR of $R_A$ = 3.35 dB. Dephasing one channel (whose contribution is approximately 0.45 dB to the net SSNR) by 50 microseconds will degrade the SSNR by 0.2 dB.
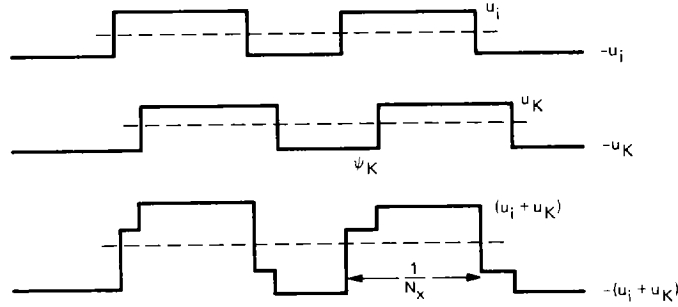


Fig. B1. Two combined channels dephased by $\Psi_K$